An alternative S-matrix for $\mathcal{N} = 6$ Chern-Simons theory?

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ABSTRACT: We have recently proposed an S-matrix for the planar limit of the $\mathcal{N} = 6$ superconformal Chern-Simons theory of Aharony, Bergman, Jafferis and Maldacena which leads to the all-loop Bethe ansatz equations conjectured by Gromov and Vieira. An unusual feature of this proposal is that the scattering of A and B particles is reflectionless. We consider here an alternative S-matrix, for which $A - B$ scattering is not reflectionless. We argue that this S-matrix does not lead to the Bethe ansatz equations which are consistent with perturbative computations.

Contents

1 Introduction

The fact that the 3-dimensional $\mathcal{N} = 6$ superconformal Chern-Simons (CS) theory of Aharony, Bergman, Jafferis and Maldacena [1] has a planar limit suggests that it may have further features in common with 4-dimensional $\mathcal{N} = 4$ superconformal Yang-Mills (YM) theory. Indeed, it was shown by Minahan and Zarembo [2] (see also [3]) that the two-loop anomalous dimensions of the scalar operators in planar $\mathcal{N} = 6$ CS theory are described by a certain integrable spin chain. Furthermore, they conjectured two-loop Bethe ansatz equations (BAEs) for the full theory. Gromov and Vieira [4] subsequently conjectured all-loop BAEs, which reduce to those of Minahan and Zarembo in the weak-coupling limit. Recently, three groups [5–7] computed the one-loop correction to the energy of a folded spinning string, and seemed to find disagreement with the prediction of the all-loop BAEs. This controversy may be resolved by a non-zero one-loop correction in the central interpolating function $h(\lambda)$ as suggested recently in [8]. (See also [9].)

Based on the spectrum and symmetries of the model [2, 10–12], we proposed an all-loop *S*-matrix [13] which reproduces the all-loop BAEs. That *S*-matrix has the unusual feature that the scattering of A and B particles is reflectionless,

$$A \, B \to B \, A$$

(instead of $A \, B \to B \, A + A \, B$). Given the uncertainty in these all-loop proposals, one may well wonder whether there exists another *S*-matrix which

(i) is not reflectionless; and

(ii) is consistent with the two-loop BAEs of Minahan and Zarembo [2], which are on firmer ground.

This note is an effort to address this question. Unfortunately, we do not give a definitive answer. Nevertheless, our failure to find such an alternative *S*-matrix provides increased confidence in our original proposal [13], and in the corresponding all-loop BAEs [4].

The outline of this paper is as follows. In section 2 we construct a candidate alternative S-matrix. The key feature of this S-matrix which allows for reflection is that it factorizes into the product of a nontrivial flavor part and an SU(2|2) part. In order to simplify the ensuing analysis, we make the unphysical assumption that the flavor part is SU(2)-invariant. (We later argue that this simplifying assumption does not alter the main conclusion.) In section 3 we derive the corresponding all-loops BAEs by diagonalizing the Bethe-Yang matrix. We perform the weak-coupling limit, and show that the result is not consistent with the two-loop BAEs [2]. We conclude in section 4 with a brief discussion of our results.

2 S-matrix

We represent the elementary excitations by Zamolodchikov-Faddeev operators $A^\dagger_{a\,i}(p)$, where $a \in \{1, 2\}$ is a flavor index ($a = 1$ corresponds to an A-particle, and $a = 2$ corresponds to a B-particle), and $i \in \{1, 2, 3, 4\}$ is the SU(2|2) index. When acting on the vacuum state $|0\rangle$, these operators create corresponding asymptotic particle states of momentum p and energy E given by [10–12, 14]

$$E = \sqrt{\frac{1}{4} + 4g^2 \sin^2 \frac{p}{2}}\,, \qquad (2.1)$$

where g is a function of the 't Hooft coupling

$$g = h(\lambda)\,, \qquad (2.2)$$

with $h(\lambda) \sim \lambda$ for small λ, and $h(\lambda) \sim \sqrt{\lambda/2}$ for large λ.

A way to allow for reflection of A and B particles, while still maintaining integrability, is to assume that the S-matrix factorizes into the product of a nontrivial flavor part and an SU(2|2) part,

$$A^\dagger_{a\,i}(p_1)\, A^\dagger_{b\,j}(p_2) = S_0(p_1, p_2)\, S^{a'\,b'}_{a\,b}(p_1, p_2)\, \widehat{S}^{i'\,j'}_{i\,j}(p_1, p_2)\, A^\dagger_{b'\,j'}(p_2)\, A^\dagger_{a'\,i'}(p_1)\,, \qquad (2.3)$$

where both the flavor S-matrix $S^{a'\,b'}_{a\,b}(p_1, p_2)$ and the SU(2|2) S-matrix $\widehat{S}^{i'\,j'}_{i\,j}(p_1, p_2)$ satisfy the Yang-Baxter equation (YBE), and $S_0(p_1, p_2)$ is an unknown scalar factor.

The SU(2|2) part is essentially fixed [15, 16], with g given by (2.2). More precisely, in order to carry out the asymptotic Bethe ansatz analysis below, we assume that $\widehat{S}^{i'\,j'}_{i\,j}(p_1, p_2)$ is the graded version [17] of the SU(2|2)-invariant S-matrix given in [18].

Since the only known symmetry relating A and B particles is CP symmetry, the flavor S-matrix should not have any more symmetry. Solutions of the YBE with only discrete symmetry are known, such as the R-matrix of the XYZ spin chain/8-vertex model; and in principle, we could proceed by assuming that the flavor S-matrix is of that form. However, in order to simplify the ensuing analysis, we instead make the unphysical assumption that the flavor S-matrix is SU(2)-invariant. We shall later argue that this simplifying assumption does not alter the main conclusion.

As is well-known (see, e.g., [19]), SU(2) symmetry and factorizability almost completely fix the structure of the S-matrix. Indeed, SU(2) symmetry implies that, up to an overall

scalar factor,

$$S_{a\,b}^{a'\,b'}(p_1, p_2) = i\delta_a^{b'}\delta_b^{a'} + f(p_1, p_2)\delta_a^{a'}\delta_b^{b'}\,, \tag{2.4}$$

where $f(p_1, p_2)$ is an arbitrary scalar function of p_1, p_2. The YBE

$$S_{12}(p_1, p_2)\, S_{13}(p_1, p_3)\, S_{23}(p_2, p_3)\ = S_{23}(p_2, p_3)\, S_{13}(p_1, p_3)\, S_{12}(p_1, p_2) \tag{2.5}$$

then implies that

$$f(p_1, p_2) = f(p_1, p_3) - f(p_2, p_3)\,, \tag{2.6}$$

which in turn implies that

$$f(p_1, p_2) = \alpha(p_1) - \alpha(p_2)\,, \tag{2.7}$$

where $\alpha(p)$ is an arbitrary function of p. In the weak-coupling limit, $\alpha(p)$ must be a linear function of p, say

$$\alpha(p) = p\,, \quad \text{(weak coupling)} \tag{2.8}$$

in order that $f(p_1, p_2)$ be a function of $p_1 - p_2$, i.e., that the S-matrix have the "difference" property. We conclude that

$$S_{a\,b}^{a'\,b'}(p_1, p_2) = i\delta_a^{b'}\delta_b^{a'} + (\alpha(p_1) - \alpha(p_2))\,\delta_a^{a'}\delta_b^{b'}\,. \tag{2.9}$$

In matrix form,

$$S(p_1, p_2) = \begin{pmatrix} a(p_1, p_2) & 0 & 0 & 0 \\ 0 & b(p_1, p_2) & i & 0 \\ 0 & i & b(p_1, p_2) & 0 \\ 0 & 0 & 0 & a(p_1, p_2) \end{pmatrix}\,, \tag{2.10}$$

where

$$a(p_1, p_2) = \alpha(p_1) - \alpha(p_2) + i\,, \qquad b(p_1, p_2) = \alpha(p_1) - \alpha(p_2)\,. \tag{2.11}$$

Note that

$$b(p_1, p_2) = -b(p_2, p_1)\,. \tag{2.12}$$

The S-matrix (2.3), unlike the one which we proposed in [13], does allow for reflection in $A - B$ scattering. Examples of integrable models with S-matrices of product form include [20]. To determine the function α, one may need to impose CP symmetry between A- and B-particles which leads to a crossing relation. We shall not pursue this here since our conclusion does not depend on the explicit form of α.

3 Asymptotic Bethe ansatz

We now proceed to derive the corresponding all-loop BAEs. The analysis is similar to the one for $\mathcal{N} = 4$ YM theory [16, 17]; and as in [13], we follow closely the latter reference. We consider a set of N particles with momenta p_i ($i = 1, \ldots, N$) which are widely separated on a ring of length L'. Quantization conditions for these momenta follow from imposing periodic boundary conditions on the wavefunction. Taking a particle with momentum p_k around the ring leads to the Bethe-Yang equations

$$e^{-ip_k L'} = \Lambda(\lambda = p_k, \{p_i\}; \{\lambda_j, \mu_j, \xi_j\}), \quad k = 1, \ldots, N, \tag{3.1}$$

where $\Lambda(\lambda, \{p_i\}; \{\lambda_j, \mu_j, \xi_j\})$ are the eigenvalues of the transfer matrix

$$t(\lambda, \{p_i\}) = \Lambda_0(\lambda, \{p_i\}) \, t_{\text{SU}(2)}(\lambda, \{p_i\}) \otimes t_{\text{SU}(2|2)}(\lambda, \{p_i\}), \tag{3.2}$$

where

$$\begin{aligned}
\Lambda_0(\lambda, \{p_i\}) &= \prod_{i=1}^{N} S_0(\lambda, p_i), \\
t_{\text{SU}(2)}(\lambda, \{p_i\}) &= \text{tr}_a \, S_{a\,1}(\lambda, p_1) \cdots S_{a\,N}(\lambda, p_N), \\
t_{\text{SU}(2|2)}(\lambda, \{p_i\}) &= \text{str}_a \, \widehat{S}_{a\,1}(\lambda, p_1) \cdots \widehat{S}_{a\,N}(\lambda, p_N).
\end{aligned} \tag{3.3}$$

Hence, the eigenvalues are given by

$$\Lambda(\lambda, \{p_i\}; \{\lambda_j, \mu_j, \xi_j\}) = \Lambda_0(\lambda, \{p_i\}) \, \Lambda_{\text{SU}(2)}(\lambda, \{p_i\}; \{\xi_j\}) \, \Lambda_{\text{SU}(2|2)}(\lambda, \{p_i\}; \{\lambda_j, \mu_j\}), \tag{3.4}$$

where the SU(2) part is given by the well-known algebraic Bethe ansatz result

$$\Lambda_{\text{SU}(2)}(\lambda, \{p_i\}; \{\xi_j\}) = \prod_{i=1}^{N} a(\lambda, p_i) \prod_{j=1}^{m_0} s(\xi_j, \lambda) + \prod_{i=1}^{N} b(\lambda, p_i) \prod_{j=1}^{m_0} s(\lambda, \xi_j), \tag{3.5}$$

with

$$s(p_1, p_2) = \frac{a(p_1, p_2)}{b(p_1, p_2)} = \frac{\alpha(p_1) - \alpha(p_2) + i}{\alpha(p_1) - \alpha(p_2)}, \tag{3.6}$$

and $\{\xi_j\}$ obey the BAEs

$$\prod_{i=1}^{N} s(\xi_k, p_i) = \prod_{\substack{j=1 \\ j \neq k}}^{m_0} \frac{s(\xi_k, \xi_j)}{s(\xi_j, \xi_k)}, \qquad k = 1, \ldots, m_0. \tag{3.7}$$

In particular, due to the property (2.12), the eigenvalues at $\lambda = p_k$ are given by

$$\Lambda_{\text{SU}(2)}(\lambda = p_k, \{p_i\}; \{\xi_j\}) = \prod_{i=1}^{N} a(p_k, p_i) \prod_{j=1}^{m_0} s(\xi_j, p_k). \tag{3.8}$$

Moreover, the SU(2|2) part is given by [17]

$$
\Lambda_{\mathrm{SU}(2|2)}(\lambda, \{p_i\}; \{\lambda_j, \mu_j\}) = \prod_{i=1}^{N} \left[\frac{x^+(\lambda) - x^-(p_i)}{x^-(\lambda) - x^+(p_i)} \frac{\eta(p_i)}{\eta(\lambda)} \right] \prod_{j=1}^{m_1} \left[\eta(\lambda) \frac{x^-(\lambda) - x^+(\lambda_j)}{x^+(\lambda) - x^+(\lambda_j)} \right]
$$

$$
- \prod_{i=1}^{N} \left[\frac{x^+(\lambda) - x^+(p_i)}{x^-(\lambda) - x^+(p_i)} \frac{1}{\eta(\lambda)} \right] \left\{ \prod_{j=1}^{m_1} \left[\eta(\lambda) \frac{x^-(\lambda) - x^+(\lambda_j)}{x^+(\lambda) - x^+(\lambda_j)} \right] \prod_{l=1}^{m_2} \frac{x^+(\lambda) + \frac{1}{x^+(\lambda)} - \tilde{\mu}_l + \frac{i}{2g}}{x^+(\lambda) + \frac{1}{x^+(\lambda)} - \tilde{\mu}_l - \frac{i}{2g}} \right.
$$

$$
\left. + \prod_{j=1}^{m_1} \left[\eta(\lambda) \frac{x^+(\lambda_j) - \frac{1}{x^+(\lambda)}}{x^+(\lambda_j) - \frac{1}{x^-(\lambda)}} \right] \prod_{l=1}^{m_2} \frac{x^-(\lambda) + \frac{1}{x^-(\lambda)} - \tilde{\mu}_l - \frac{i}{2g}}{x^-(\lambda) + \frac{1}{x^-(\lambda)} - \tilde{\mu}_l + \frac{i}{2g}} \right\}
$$

$$
+ \prod_{i=1}^{N} \left[\frac{x^+(\lambda) - x^+(p_i)}{x^-(\lambda) - x^+(p_i)} \frac{1 - \frac{1}{x^-(\lambda) x^+(p_i)}}{1 - \frac{1}{x^-(\lambda) x^-(p_i)}} \frac{\eta(p_i)}{\eta(\lambda)} \right] \prod_{j=1}^{m_1} \left[\eta(\lambda) \frac{x^+(\lambda_j) - \frac{1}{x^+(\lambda)}}{x^+(\lambda_j) - \frac{1}{x^-(\lambda)}} \right], \tag{3.9}
$$

where $\eta(\lambda) = e^{i\lambda/2}$, and the corresponding BAEs are given by

$$
e^{iP/2} \prod_{i=1}^{N} \frac{x^+(\lambda_j) - x^-(p_i)}{x^+(\lambda_j) - x^+(p_i)} = \prod_{l=1}^{m_2} \frac{x^+(\lambda_j) + \frac{1}{x^+(\lambda_j)} - \tilde{\mu}_l + \frac{i}{2g}}{x^+(\lambda_j) + \frac{1}{x^+(\lambda_j)} - \tilde{\mu}_l - \frac{i}{2g}}, \qquad j = 1, \ldots, m_1,
$$

$$
\prod_{j=1}^{m_1} \frac{\tilde{\mu}_l - x^+(\lambda_j) - \frac{1}{x^+(\lambda_j)} + \frac{i}{2g}}{\tilde{\mu}_l - x^+(\lambda_j) - \frac{1}{x^+(\lambda_j)} - \frac{i}{2g}} = \prod_{\substack{k=1 \\ k \neq l}}^{m_2} \frac{\tilde{\mu}_l - \tilde{\mu}_k + \frac{i}{g}}{\tilde{\mu}_l - \tilde{\mu}_k - \frac{i}{g}}, \qquad l = 1, \ldots, m_2, \tag{3.10}
$$

where

$$
\frac{x^+(\lambda)}{x^-(\lambda)} = e^{i\lambda}, \quad x^+(\lambda) + \frac{1}{x^+(\lambda)} - x^-(\lambda) - \frac{1}{x^-(\lambda)} = \frac{i}{g}, \qquad P = \sum_{i=1}^{N} p_i. \tag{3.11}
$$

In particular, the eigenvalue at $\lambda = p_k$ is given simply by

$$
\Lambda_{\mathrm{SU}(2|2)}(\lambda = p_k, \{p_i\}; \{\lambda_j, \mu_j\}) = \prod_{i=1}^{N} \left[\frac{x^+(p_k) - x^-(p_i)}{x^-(p_k) - x^+(p_i)} \frac{\eta(p_i)}{\eta(p_k)} \right] \prod_{j=1}^{m_1} \left[\eta(p_k) \frac{x^-(p_k) - x^+(\lambda_j)}{x^+(p_k) - x^+(\lambda_j)} \right]. \tag{3.12}
$$

In view of (3.8), (3.12), the Bethe-Yang equations (3.1) take the form

$$
e^{ip_k\left(-L' + \frac{N}{2} - \frac{m_1}{2}\right)} = e^{iP/2} \prod_{i=1}^{N} \left\{ S_0(p_k, p_i) \, a(p_k, p_i) \left[\frac{x^+(p_k) - x^-(p_i)}{x^-(p_k) - x^+(p_i)} \right] \right\}
$$

$$
\times \prod_{j=1}^{m_0} s(\xi_j, p_k) \prod_{j=1}^{m_1} \frac{x^-(p_k) - x^+(\lambda_j)}{x^+(p_k) - x^+(\lambda_j)}, \quad k = 1, \ldots, N, \tag{3.13}
$$

where $\{\lambda_j, \mu_j, \xi_j\}$ are determined by the BAEs (3.7), (3.10).

Following [13, 17], we make the identifications

$$
x^{\pm}(p_k) = x^{\pm}_{4,k}, \qquad k = 1, \ldots, K_4 \equiv N,
$$

$$
x^+(\lambda_j) = \frac{1}{x_{1,j}}, \qquad j = 1, \ldots, K_1,
$$

$$
x^+(\lambda_{K_1+j}) = x_{3,j}, \qquad j = 1, \ldots, K_3, \qquad\qquad K_1 + K_3 \equiv m_1,
$$

$$
\tilde{\mu}_j = \frac{u_{2,j}}{g}, \qquad j = 1, \ldots, K_2 \equiv m_2, \tag{3.14}
$$

and also define

$$u_{4,j} = x_{4,j}^+ + \frac{1}{x_{4,j}^+} - \frac{i}{2} = x_{4,j}^- + \frac{1}{x_{4,j}^-} + \frac{i}{2}\,, \tag{3.15}$$

and $u_{i,j} = g\left(x_{i,j} + \frac{1}{x_{i,j}}\right)$ for $i = 1, 3$. We assume the zero-momentum condition

$$P = \sum_{j=1}^{K_4} p_{4,j} = 0\,, \tag{3.16}$$

and (for aesthetic reasons) we perform the shift

$$\alpha(\xi_j) \to \alpha(\xi_j) - \frac{i}{2}\,. \tag{3.17}$$

The Bethe-Yang equations (3.13) become

$$e^{i p_{4,k}\left(-L' + \frac{K_4 + K_1 - K_3}{2}\right)} = \prod_{i=1}^{K_4} \left\{ S_0(p_{4,k}, p_{4,i})\, [\alpha(p_{4,k}) - \alpha(p_{4,i}) + i]\left(\frac{x_{4,k}^+ - x_{4,i}^-}{x_{4,k}^- - x_{4,i}^+}\right)\right\}$$

$$\times \prod_{j=1}^{m_0} \frac{\alpha(\xi_j) - \alpha(p_{4,k}) + \frac{i}{2}}{\alpha(\xi_j) - \alpha(p_{4,k}) - \frac{i}{2}} \prod_{j=1}^{K_1} \frac{1 - \frac{1}{x_{4,k}^- x_{1,j}}}{1 - \frac{1}{x_{4,k}^+ x_{1,j}}} \prod_{j=1}^{K_3} \frac{x_{4,k}^- - x_{3,j}}{x_{4,k}^+ - x_{3,j}}\,, \quad k = 1, \dots, K_4\,, \tag{3.18}$$

and the BAEs (3.7), (3.10) become

$$\prod_{i=1}^{K_4} \frac{\alpha(\xi_k) - \alpha(p_{4,i}) + \frac{i}{2}}{\alpha(\xi_k) - \alpha(p_{4,i}) - \frac{i}{2}} = \prod_{\substack{j=1 \\ j \neq k}}^{m_0} \frac{\alpha(\xi_k) - \alpha(\xi_j) + i}{\alpha(\xi_k) - \alpha(\xi_j) - i}\,, \quad k = 1, \dots, m_0\,,$$

$$\prod_{i=1}^{K_4} \frac{1 - \frac{1}{x_{1,j} x_{4,i}^-}}{1 - \frac{1}{x_{1,j} x_{4,i}^+}} = \prod_{l=1}^{K_2} \frac{u_{1,j} - u_{2,l} + \frac{i}{2}}{u_{1,j} - u_{2,l} - \frac{i}{2}}\,, \quad j = 1, \dots, K_1\,,$$

$$\prod_{i=1}^{K_4} \frac{x_{3,j} - x_{4,i}^-}{x_{3,j} - x_{4,i}^+} = \prod_{l=1}^{K_2} \frac{u_{3,j} - u_{2,l} + \frac{i}{2}}{u_{3,j} - u_{2,l} - \frac{i}{2}}\,, \quad j = 1, \dots, K_3\,,$$

$$\prod_{j=1}^{K_1} \frac{u_{2,l} - u_{1,j} + \frac{i}{2}}{u_{2,l} - u_{1,j} - \frac{i}{2}} \prod_{j=1}^{K_3} \frac{u_{2,l} - u_{3,j} + \frac{i}{2}}{u_{2,l} - u_{3,j} - \frac{i}{2}} = \prod_{\substack{k=1 \\ k \neq l}}^{K_2} \frac{u_{2,l} - u_{2,k} + i}{u_{2,l} - u_{2,k} - i}\,, \quad l = 1, \dots, K_2\,, \tag{3.19}$$

respectively. Eqs. (3.18), (3.19) constitute our result for the all-loop BAEs corresponding to the S-matrix (2.3), (2.9).

The weak-coupling limit corresponds to [4]

$$x \to \frac{u}{g}\,, \qquad x^\pm \to \frac{1}{g}\left(u \pm \frac{i}{2}\right)\,, \tag{3.20}$$

with $g \to 0$ and u finite. Recalling (2.8), we obtain

$$\left(\frac{u_{4,k}+\frac{i}{2}}{u_{4,k}-\frac{i}{2}}\right)^L = \prod_{i=1}^{K_4}\left\{S_0(p_{4,k},p_{4,i})\,(p_{4,k}-p_{4,i}+i)\left(\frac{u_{4,k}-u_{4,i}+i}{u_{4,k}-u_{4,i}-i}\right)\right\}$$

$$\times \prod_{j=1}^{m_0}\frac{\xi_j-p_{4,k}+\frac{i}{2}}{\xi_j-p_{4,k}-\frac{i}{2}}\prod_{j=1}^{K_3}\frac{u_{4,k}-u_{3,j}-\frac{i}{2}}{u_{4,k}-u_{3,j}+\frac{i}{2}}, \qquad k=1,\dots,K_4\,,$$

$$1 = \prod_{\substack{j=1\\j\neq k}}^{m_0}\frac{\xi_k-\xi_j+i}{\xi_k-\xi_j-i}\prod_{i=1}^{K_4}\frac{\xi_k-p_{4,i}-\frac{i}{2}}{\xi_k-p_{4,i}+\frac{i}{2}}, \qquad k=1,\dots,m_0\,,$$

$$1 = \prod_{l=1}^{K_2}\frac{u_{1,j}-u_{2,l}+\frac{i}{2}}{u_{1,j}-u_{2,l}-\frac{i}{2}}, \qquad j=1,\dots,K_1\,,$$

$$1 = \prod_{l=1}^{K_2}\frac{u_{3,j}-u_{2,l}+\frac{i}{2}}{u_{3,j}-u_{2,l}-\frac{i}{2}}\prod_{i=1}^{K_4}\frac{u_{3,j}-u_{4,i}-\frac{i}{2}}{u_{3,j}-u_{4,i}+\frac{i}{2}}, \qquad j=1,\dots,K_3\,,$$

$$\text{(3.21)}$$

$$1 = \prod_{\substack{k=1\\k\neq l}}^{K_2}\frac{u_{2,l}-u_{2,k}-i}{u_{2,l}-u_{2,k}+i}\prod_{j=1}^{K_1}\frac{u_{2,l}-u_{1,j}+\frac{i}{2}}{u_{2,l}-u_{1,j}-\frac{i}{2}}\prod_{j=1}^{K_3}\frac{u_{2,l}-u_{3,j}+\frac{i}{2}}{u_{2,l}-u_{3,j}-\frac{i}{2}}, \qquad l=1,\dots,K_2\,,$$

where we have defined

$$L = -L' + \frac{K_4+K_1-K_3}{2}\,, \qquad (3.22)$$

and used

$$e^{ip_{4,k}} = \frac{u_{4,k}+\frac{i}{2}}{u_{4,k}-\frac{i}{2}}\,. \qquad (3.23)$$

Evidently, regardless of the choice of scalar factor $S_0(p_1,p_2)$, the set of BAEs (3.21) does not completely match any of the equivalent sets of BAEs proposed by Minahan and Zarembo [2]. In particular, while the latter have two "massive" nodes, the former has only one. We would have obtained a similar result had we chosen the flavor S-matrix to be of the XYZ chain/8-vertex model form rather than (2.9). We conclude that an S-matrix of the form (2.3) is not consistent with the perturbative BAEs [2].

4   Discussion

We have considered an alternative S-matrix for $\mathcal{N}=6$ CS which is symmetric under SU(2|2). In contrast with our original proposal [13], this S-matrix has the tensor product form (2.3); and it has not only an SU(2|2) part, but also a nontrivial flavor part which allows for reflection in $A-B$ scattering. Although we have not proved that this tensor product structure is the only possible way of introducing reflection while both maintaining integrability and respecting the system's symmetry, we have not found any other. We have argued that such an S-matrix is not consistent with the perturbative BAEs [2]. This gives

increased confidence in our original proposal [13]. Further support for the proposal [13] has recently been found in computations of finite-size corrections to the dispersion relation of giant magnons [21, 22], and in the direct coordinate Bethe ansatz computation of the two-loop scalar-sector S-matrix [23].

Acknowledgments

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